Exercise 13

For the following exercises, sketch the parametric equations by eliminating the parameter. Indicate any asymptotes of the graph.

$$x = 3 - 2\cos\theta, \quad y = -5 + 3\sin\theta$$

Solution

Solve each of the equations for $\cos\theta$ and $\sin\theta$.

$$\frac{3-x}{2} = \cos\theta, \quad \frac{y+5}{3} = \sin\theta$$

Square both sides of each equation and add the respective sides together.

$$\left(\frac{3-x}{2}\right)^2 + \left(\frac{y+5}{3}\right)^2 = \cos^2\theta + \sin^2\theta$$

$$\frac{(3-x)^2}{4} + \frac{(y+5)^2}{9} = 1$$

This is an ellipse centered at $(3, -5)$ with the major axis in the y-direction and the minor axis in the x-direction. Below is a plot of the parametric equations for $0 \le t \le 2\pi$.